OPTIMAL REPLACEMENT STRATEGY USING RENEWAL THEORY AND WEIBULL DISTRIBUTION by Solly Mokoena – Reliability Engineer at SASOL MINING Ltd.

ABSTRACT

This paper looks at an alternative method that can be used to solve the replacement point dilemma. The author uses the concept of Weibull distribution and the renewal theory as an alternative to practices of averaging and ‘gut feel’. To make the study practical, the author has randomly chosen a subassembly of the continuous miner used at Sasol Mining Ltd. The theoretical background has been covered slightly but very comprehensively.

1. INTRODUCTION

Performing the predetermined maintenance according to a cost optimal schedule can imply large savings and increased competitiveness for many companies. In the mining industry maintenance practices, and at Sasol Mining in particular, many decisions such as replacing expensive subassemblies, involve the investment of large sums of money. The cost and benefits accruing from the investment will continue for a number of years. When the investment of money today influences cash flows in the future and when evaluating the alternative investment opportunities, account ought to be taken of the fact that today’s value of a certain amount of money depends on when that amount is due. Given the need to minimise running cost, the use of gut feel to estimate repair/replace points can have negative impacts on future cash flows. The paper to be presented is to show how one can use a combination of statistical methods and the concept of present value of money (LCC) to determine the optimum replacement time.

The primary purpose of this paper is to share the knowledge of renewable system with maintenance practitioners in the mining industry to get rid of a gut feel approach. To accomplish this the author examines subassemblies data on a Sasol Mining Continuous Miner with random lifetime.

Upon failure the maintenance team takes a maintenance action, specifying the degree of repair. Using the data captured in the SAP system and using methods prescribed by Jardine(1992:31-91), it is possible to determine an age-dependent maintenance strategy which minimises the total **expected discounted cost** over an infinite planning horizon . Using several properties of the optimal policy which are derived in this paper, analytical and numerical methods for determining the optimal maintenance strategy are proposed.

In order to obtain a better insight regarding the structure and nature of the optimal policy and to illustrate computational procedures, a numerical example is analysed using real raw data from Sasol Mining Equipment History. The proposed maintenance model demonstrate powerful techniques and channels in the area of reliability with interesting theoretical issues and a wide range of potential applications in various aspect such as budgeting and life cycle cost management.

1.1 OBJECTIVES

The primary objectives of this paper are as follows:

- To calculate the optimal lives of subassemblies using the renewable theory
- To determine the Life Cycle Cost (LCC) implications for both the current paradigm and the scientific approach (Time Value of Money)
- To determine the failure and reliability profiles of the various subassemblies
- To determine the failure characteristics of the various subassemblies

Using the Weibull distribution and the time value of money concept to determine the optimal life of subassemblies on a continuous miner (CM) of Sasol Mining Ltd.

Published by Solly Mokoena (Reliability Engineer at Sasol Mining Ltd). – B. Sc. Mechanical Engineering (Wits University) with permission granted by Sasol Mining Ltd.

1.2 SCOPE

The study in this paper is based on the equipment pool of Sasol Mining Limited. Although the pool of equipment at Sasol Mining Ltd. is very large, the current study focussed mainly on the sub-units used on the, continuous miner (CM), one of Sasol Mining's primary production equipment. In all cases, the data used is stored on the Sasol Mining's SAP information management system under Business Warehouse. The work in this report will cover the following:

- Systematic tabulation of raw data
- Brief background of the theory to be deployed
- Data processing using actual raw data
- Summarised result
- Net Present Value of the expected savings achievable.

2. THEORETICAL BACKGROUND

2.1 Weibull Reliability- Jardine(1992:16-21)

The general Weibull distribution curve is

$$R = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

where R is the probability of surviving through time t, beta is the shape factor and eta is the scale factor.

Cumulative Distribution Function (cdf):	$F(t) = 1 - R(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$
Probability Distribution Function (pdf):	$f(t) = \frac{dF}{dt} = \frac{\beta}{\eta^{\beta}} t^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$
Hazard Rate:	$h(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\eta^{\beta}} t^{\beta-1}$

The value of beta determines the shape of the Weibull curve. For example, beta=1 corresponds to a constant hazard rate or an exponential distribution. Beta>1 means that the hazard rate increases with increasing age. This is summarised in figure 2.1 below:

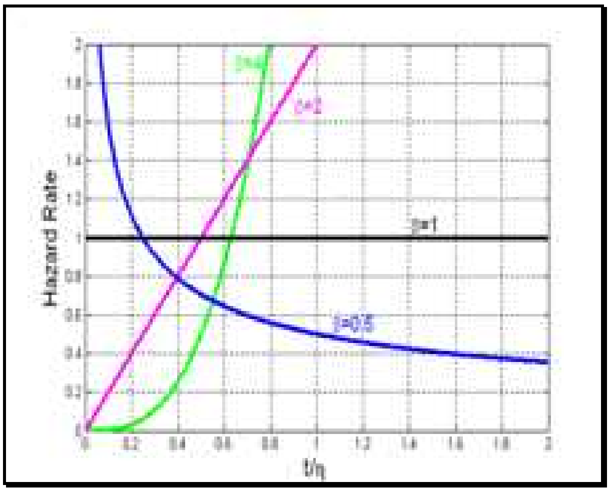

Figure 2.1: Effect of shape Weibull factor

2.2 THE RENEWAL THEORY

The time to fail for an item is variable, and can be represented by a probability distribution, $f(x)$. Referring to Figure 2.2 , the cost of failures per unit time decreases as preventive maintenance is done more often, but the cost of preventive maintenance per unit time increases. There exists a point where the total cost of failures and preventive maintenance per unit time is at a minimum; the optimum schedule for preventive maintenance Brick et al (1995:55-60)

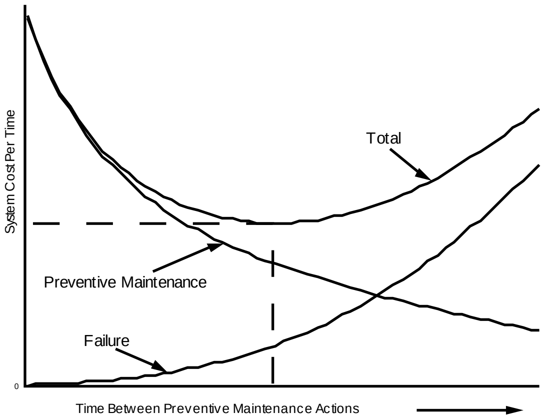

Figure 2.2: Optimal cost of replacement

The optimum time between maintenance actions is found by minimizing the total cost per unit time, Jardine(1973:90).

$$C_T = \frac{C_p R(t) + C_f (1 - R(t))}{(t_p) R(t) + \int_{\infty}^{t_p} f(t) dt}$$

where C_p is the cost of preventive maintenance, C_f is the cost of a failure and tp is the time between preventive maintenance actions.

Minimizing the equation above is tedious, and numerical routines will be used with the following assumptions:

- The time to failure follows a Weibull distribution.
- Preventive maintenance is performed on an item at time *Tp* at a cost of C_p.
- If the item fails before time = *Tp*, a failure cost of C_f is incurred.
- Each time preventive maintenance is performed, the item is returned to its initial state; that is, the item is "as good as new."

Furthermore, R(t) is the reliability function of the component, i.e. the probability that the component survives the time interval [0, t] and that it is still working at time t.

3. PROPOSED STRATEGY

Data regarding the failure data of subassemblies was obtained from the SAP Business Warehouse. Although this data goes back a few years, the author only utilised data from the previous calendar year. As mentioned earlier, cost data was hypothesised. The raw data together with the fictitious cost was processed using discrete methods on Excel spreadsheet. The component that was analysed is one of the key motors on the continuous miner system. The following procedure was then followed. All figures given in the analysis were normalised for to afford the reader a holistic picture.

- **Used Weibull Distribution** to analyse equipment failure characteristics, reliability profiles and spreading out the cost on a continuous distribution

- **Used Renewal theory** of electromechanical subsystems to determine optimal cost for a specific tonnage carried by each subassembly

- **Used Net Present Concept** to determine **Life Cycle Cost** over a period of 12 cycles

4. RAW DATA AND DATA PROCESSING

Table 4.1 : Ranking of failure data – Traction motor

Life/annual production	Ranking	Median Ranking	1/(1-Mediad Rank)	Ln(ln(1/1-Med Rank))	Ln(Life)
0.0318	1	0.0126	1.0128	-4.3649	10.3147
0.0572	2	0.0307	1.0317	-3.4684	10.9022
0.1015	3	0.0487	1.0512	-2.9964	11.4770
0.1083	4	0.0668	1.0716	-2.6719	11.5416
0.1624	5	0.0848	1.0927	-2.4230	11.9463
0.1872	6	0.1029	1.1147	-2.2203	12.0888
0.1964	7	0.1209	1.1376	-2.0487	12.1365
0.2076	8	0.1390	1.1614	-1.8995	12.1921
0.2484	9	0.1570	1.1863	-1.7671	12.3716
0.2505	10	0.1751	1.2123	-1.6478	12.3798
0.2790	11	0.1931	1.2394	-1.5390	12.4878
0.2907	12	0.2112	1.2677	-1.4387	12.5287
0.3439	13	0.2292	1.2974	-1.3456	12.6968
0.3446	14	0.2473	1.3285	-1.2585	12.6988
0.4042	15	0.2653	1.3612	-1.1765	12.8585
0.4071	16	0.2834	1.3955	-1.0989	12.8656
0.4389	17	0.3014	1.4315	-1.0252	12.9407
0.4443	18	0.3195	1.4695	-0.9547	12.9529
0.4456	19	0.3375	1.5095	-0.8872	12.9559
0.4811	20	0.3556	1.5518	-0.8223	13.0326
0.4860	21	0.3736	1.5965	-0.7596	13.0426
0.5368	22	0.3917	1.6439	-0.6990	13.1420
0.5390	23	0.4097	1.6942	-0.6402	13.1462
0.5613	24	0.4278	1.7476	-0.5829	13.1867
0.5634	25	0.4458	1.8046	-0.5271	13.1905
0.6139	26	0.4639	1.8653	-0.4725	13.2763
0.6175	27	0.4819	1.9303	-0.4190	13.2821
0.6633	28	0.5000	2.0000	-0.3665	13.3537
0.6867	29	0.5181	2.0749	-0.3148	13.3884
0.7144	30	0.5361	2.1556	-0.2639	13.4278
0.7419	31	0.5542	2.2429	-0.2135	13.4657
0.7431	32	0.5722	2.3376	-0.1636	13.4673
0.8072	33	0.5903	2.4405	-0.1140	13.5501
0.8112	34	0.6083	2.5530	-0.0648	13.5549
0.8349	35	0.6264	2.6763	-0.0157	13.5838
0.8373	36	0.6444	2.8122	0.0334	13.5866
0.8405	37	0.6625	2.9626	0.0826	13.5904
0.9411	38	0.6805	3.1299	0.1319	13.7035
1.0600	39	0.6986	3.3174	0.1816	13.8225

As shown in the table above, raw data of a traction motor, one of the major components of a CM, was collected. In the table above, the data was ranked using the method of median ranking in ascending order.

The most important thing about organising data as in the previous section is to calculate the values of the shape factor, β, and the characteristic life, η. These are calculated using numerical methods and linear regression.

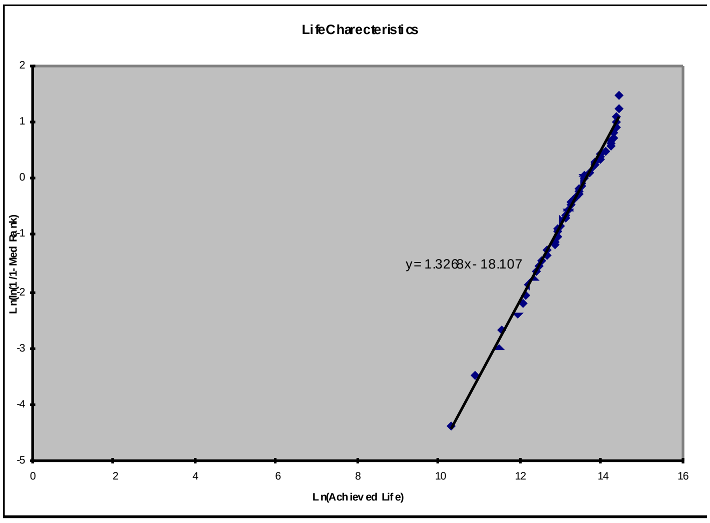

Figure 4.1: Determination of the β and η.

4.1 PROCESSED DATA - RESULTS

After performing a linear regression, the value of the shape factor worked out to be 1.3268 while the characteristic life worked out to be 89% of the annual production. These could also be seen from the straight-line equation on the graph in figure 4.1 above.

Using this information, a reliability profile was generated using the Weibull reliability function:

$$R = e^{-\left(\frac{t}{\eta}\right)^{\beta}}.$$

The resulting reliability distribution function is as shown in the figure below.

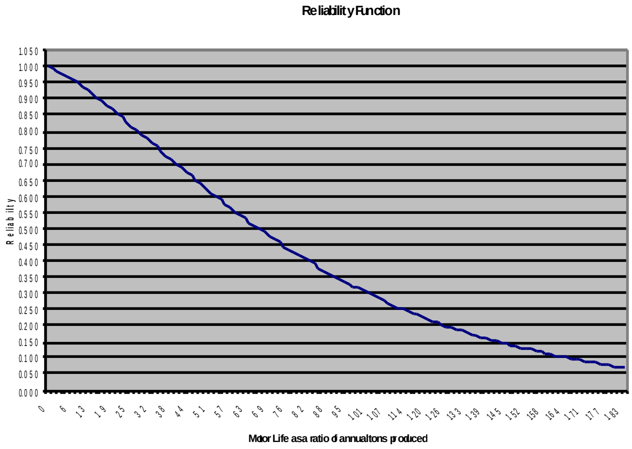

Figure 4.2: The survival probability of a traction motor.

The value of $\beta = 1.3268$ implies that failure occurs due to fatigue related problems, i.e. in the wear and tear region of the **Bathtub curve**. The value of the characteristic life, $\eta = 89\%$ of annual production. This implies that 63.2% of the selected motors reach this life. One can therefore conclude that the MTTF of this subassembly is 89% of the annual production tons. This is not however the optimal cost point.

4.2 Optimal Cost Point

Using the renewal theory equation and the numerical methods, the figure 4.3 was obtained and it is in line with the hypotheses mentioned earlier.. At the optimal point, the unit cost will be 5.03% of the total maintenance cost of the CM.

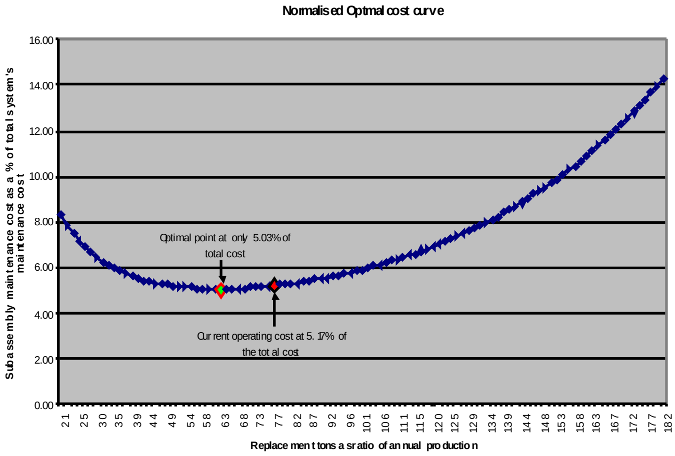

Figure 4.3: Normalised Optimal Cost Curve

The graph also shows the cost of not operating at optimal replacement life. Looking at the current paradigm, the maintenance cost on the motor is at 5.17% of the total maintenance. A simple conclusion is that the maintenance cost would drop by 0.14% per replacement. What are the Life Cycle Cost implications of this outcome?

If one ignores inflation and assumes constant cash outflow, one can use the NPV equation to calculate the net present savings.

Under these conditions, the, $NPV = \frac{C(1-(\frac{1}{1+WACC})^{n}}{WACC}$.

Where NPV = net present value, C = annual maintenance cost, WACC = 15% = weighted average cost of capital and n = number of years =12 years.

As shown in the decision tree in figure 4.4, the expected cost, NPV, under this philosophy will be at 0.59 % of the total maintenance cost , whilst in the current practice it would be at 0.61 of the total maintenance cost on the continuous miner.

This results in a savings of 0.2% per annum per annum for two motors per machine.

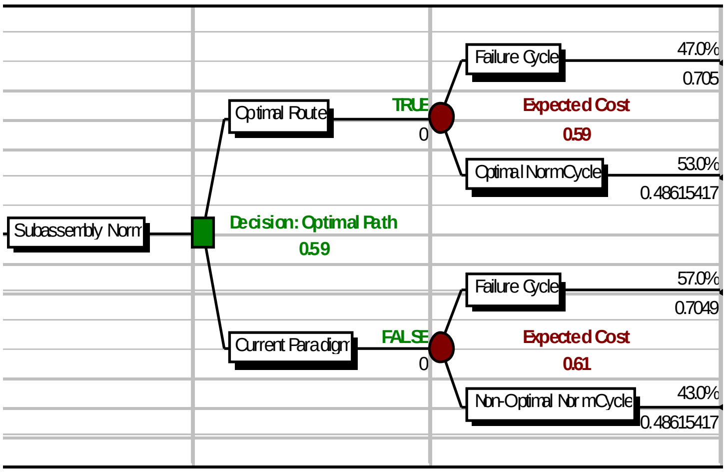

Figure 4.4 Decision tree – Minimal cost increase route chosen

5. CONCLUSION

- The optimum route has proven to be rewarding in terms of the expected per unit maintenance cost. This also implies an even bigger savings on the total life cycle cost of the entire system.

- Analytical methods yield can be used to plan preventative maintenance intervals

- The value of beta or shape factor can also be used describe the failure pattern of equipment.

6. RECOMMENDATIONS

It is recommended that, the reader should use the model on other subsystems and compare the result with their current replacement philosophy.

REFERENCES

Brick, M, Michael, J., and Morganstein, D. (May 1989). "Using Statistical Thinking to Solve Maintenance Problems," *Quality Progress*, pp. 55 - 60.

Jardine AKS. 1992, Maintenance, Replacement & Reliability, *Pitman Printing*, Toronto, pp. 6 - 91